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An Innovative Approach to Radiality Representation in Electrical Distribution System Reconfiguration: Enhanced Efficiency and Computational Performance

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**Abstract:** The reconfiguration problem in electrical distribution systems is a critical area of research, aimed at optimizing the operational efficiency of these networks. Historically, this problem has been approached through a variety of optimization approaches. Regarding mathematical models, a key challenge identified in these models is the formulation of equations that ensure the radial operation of the system along with the nonlinear equations representing Kirchhoff's laws, the last often necessitating complex relaxations for practical application. This paper introduces an alternative representation of system radiality, which potentially surpasses or matches existing methods in literature. Our approach utilizes a more intuitive and compact set of equations, simplifying the representation process. Additionally, we propose a linearization of the current calculation in the power flow model originally proposed by Wu. This linearization significantly accelerates the process of obtaining feasible solutions and optimal reconfiguration profiles. To validate our approach, we conducted rigorous computational comparisons with results reported in existing literature, using a variety of test cases to ensure robustness. Our computational results demonstrate a considerable improvement in computational time. The objective functions used are competitive and, in many instances, outperform the best-reported results in the literature. In some cases, our method even identifies superior solutions.

**Keywords:** Power Distribution System Reconfiguration; Radial operation, Mathematical Modeling, Linearization Techniques, Computational Efficiency.

1. Introduction

The electric power distribution system, a cornerstone of modern infrastructure, is undergoing a critical transformation to meet the escalating energy demands and the growing emphasis on sustainability and reliability. At the heart of this transformation is the reconfiguration problem, a process integral to optimizing network performance and enhancing operational efficiency [1]. Reconfiguration involves altering the topology of the power distribution network, primarily to reduce losses, improve load balancing, and enhance fault response mechanisms. This process is not only pivotal for the efficient functioning of power grids but also for integrating renewable energy sources and adapting to the dynamic nature of modern energy demands [2].

The reconfiguration problem, however, is inherently complex due to the need to maintain a radial network structure in operation mode. Radial networks are preferred in power distribution due to their operational simplicity and ease of fault management. Yet, the maintenance of this radial structure during reconfiguration presents a significant challenge, compounded by the nonlinear nature of the underlying power flow equations, as dictated by Kirchhoff's laws [3]. The complexity of these equations often necessitates simplifications or approximations, which can compromise the accuracy and effectiveness of the reconfiguration process.

The current state of research is characterized by a plethora of mathematical models aiming to address these challenges. However, these models often diverge in their approach to representing radiality and managing the nonlinear equations involved. Some models prioritize computational simplicity at the cost of accuracy, while others aim for high fidelity but face computational intractability [4], [5], [6]. This divergence has led to a vibrant but fragmented research landscape, with no clear consensus on the optimal approach to tackle the reconfiguration problem.

Considering these challenges, our study proposes a novel approach to the reconfiguration problem. We introduce an alternative representation of radiality that promises to be more intuitive and involve a smaller set of equations than currently existing models. This approach not only simplifies the mathematical representation but also aims to improve the computational efficiency of the reconfiguration process. Additionally, we present a linearization technique for the current calculations in the power flow model, initially proposed by Baran and Wu [1], to expedite the derivation of feasible solutions.

The primary aim of this work is to bridge the gap between the need for accurate representation of radial networks and the computational challenges posed by the reconfiguration problem. Our findings demonstrate a marked improvement in processing time and solution quality, offering a promising new direction in the field of power distribution system optimization [2], [3]. The study's conclusions are significant not only for their technical contributions but also for their potential to guide future research and practical implementations in an area crucial to the evolving landscape of power distribution networks.

2. Literature Review

The electric power distribution system reconfiguration (DPSR) problem has been a subject of intensive research due to its critical role in optimizing network efficiency and reliability. This review will focus on key developments in the field, particularly in the context of mathematical modeling and algorithmic solutions for DPSR.

Mathematical optimization methods enable the straightforward resolution of linear optimization problems, ensuring convergence to the global optimum [7]; however, their drawback when addressing combinatorial optimization problems with large search spaces is that they are computationally expensive and, in some cases, impractical in a real application. Despite these limitations, the enhancement of computing capabilities, coupled with the development of quantum computing [8] sustains the ongoing interest of researchers in this field.

This literature review delves into the critical advancements in DPSR, with a particular focus on the mathematical programming techniques and the representation of radiality, which are essential for accurately modeling and solving these problems.

The foundational work in DPSR dates to Merlin and Back in 1975 [4], who introduced a network loss model based on branch resistances and current magnitudes. This early model, while foundational, lacked the sophistication needed for larger and more complex networks.

The first study on the network reconfiguration problem was conducted by A. Merlin and H. Back in 1975 [4]. Losses in the network were modeled by the well-known Joul’s effect. The solution scheme begins by calculating power flow in the network, considering all circuit breakers as closed. The solution is iteratively updated by opening circuit breakers in feeders with the least power flow until achieving a radial topology where the number of feeders equals the number of nodes minus one. While convergence of the reconfiguration problem was ensured, the iterative process was slow [7].

In 1989, C. Liu and S. Lee considered the distribution system loads as current sinks. With this model, they transformed the network reconfiguration problem into a quadratic programming problem that was solved by linearizing the objective function [9]. While the results obtained with this technique were promising, the model was computationally expensive [5].

In 1990, Glamocanin et al. formulated the network reconfiguration problem as a transportation problem [10] with quadratic costs due to the dependency of losses on the square of the current. Starting from an optimal initial configuration obtained by linearizing power losses, the configuration was iteratively updated using the quadratic SIMPLEX method. The proposed technique has the drawback that the quadratic SIMPLEX method tends to find local optima [5].

In 1991, Wagner et al. utilized an algorithm to solve the network reconfiguration problem based on a linear programming solution to the transportation problem. Power losses in the feeders were approximated using linear functions. Although the algorithm is computationally efficient for small networks, for networks with more than 1000 nodes, the solution time renders it impractical in a real DMS [5].

In 2012, Lavorato et al. made a fundamental contribution to the field of reconfiguration by proposing more precisely the radiality constraints of the network [11]. Thanks to this advancement, new models emerged in literature. In the same year, Jabr et al. formulated the reconfiguration problem as a Mixed-Integer Conic Programming (MICP) problem [6]. While the results obtained were identical to those achieved with Mixed-Integer Linear Programming, the effort required to formulate the problem constraints in polyhedral representation was significantly higher.

Hover et al. employed Mixed-Integer Quadratic Programming (MIQP), Quadratically Constrained Programming (QCP), and Second-Order Cone Programming (SOCP) to address the reconfiguration problem. Among the three studied techniques, MIQP exhibited better efficiency for networks with a large number of nodes [12].

In 2014, Marlon et al. proposed a method based on Mixed-Integer Linear Programming [3] by linearizing the equation that relates currents, voltages, and active and reactive powers. With this technique, the convergence of the problem was ensured, and the same solutions reported as the best in literature were found.

In 2015, Haghigat et al. employed a Mixed-Integer Linear Programming model, including a two-stage decomposition algorithm [13]. While the decomposition algorithm was able to solve the reconfiguration problem, the linear approximations used diminished the precision of the solution [14].

In recent times, the integration of microgrids and distributed generation sources has marked some of the research trends in this field. In 2020, Wang et al. addressed the reconfiguration problem with the aim of maximizing generation from renewable sources [15]. They employed a Markov Decision Process (MDP) in which the system states are determined by the various levels of generation from distributed sources and the network topology. In 2021, Mahdavi et al. proposed a second-order cone integer programming model [7]. The obtained results demonstrate reduced computational time compared to other techniques, although the applicable systems are of medium size.

Pareja et al. in 2022, employed a mathematical model that closely resembled the one described by [3] incorporating the presence of distributed generation in the network, along with the optimization problem of strategically placing such distributed generation. Experiments were conducted on a system with 202 nodes, achieving lower computational times compared to those reported in literature using alternative algorithms [2]. Furthermore, the identified topologies were superior to those documented in the literature.

Other techniques such as heuristics, metaheuristics, machine learning etc. have been widely used to solve the DPSR due to its reduced computational time; In **Table 1**, some of the key works reported in literature on different techniques for solving the DPSR are presented. For a comprehensive list, references [5] and [7] can be consulted.

**Table 1.** Relevant works developed in literature divided into the main categories: optimization models, heuristics, metaheuristics, and artificial intelligence.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Ref.** | **Year** | **Author(s)** | **Model description** |
| ***Mathematical optimization models*** | [4] | 1975 | Merlin y Back | Integer programming model. Modeling losses with the equation where n corresponds to the number of branches. |
| [9] | 1989 | C. Liu y S. Lee | Quadratic Programming Model – Linearization of the Objective Function |
| [5] | 1991 | Wagner et. al | Algorithm based on the linear transportation problem. Efficient for networks. pequeñas. |
| [16] | 1996 | Abur | Model for the minimum cost of active load. |
| [17] | 2005 | Ramos et. al | Genetic algorithm and mixed-integer linear programming model. |
| [18] | 2010 | Romero-Ramos et. al | Mixed-integer quadratic programming model. |
| [6] | 2012 | Jabr et. al | Mixed-integer conic linear programming model - Mixed-integer linear programming model (polyhedral representation of conic constraints). |
| [19] | 2014 | Marlon C.O. Borges et. al | Method based on MILP, involving the linearization of the equation relating currents, voltages, and active and reactive powers. |
| [2] | 2022 | Pareja et. al | Method based on mixed-integer linear programming with distributed generation in the network. |
| ***Heuristics*** | [1] | 1989 | Baran y Wu | Branch switching model and application of flow calculation methods. |
| [20] | 1992 | Goswami y Basu | Iterative model of branch closure and opening until finding the one with the least load loss. |
| [21] | 2008 | Raju y Bijwe | Iterative model of branch closure and opening until finding the most efficient configuration - initially closed switches. |
| [22] | 2011 | Abul'Wafa | Branch switching model - power flow algorithm with graph theory. |
| [23] | 2020 | Zhan et al. | Stochasticity of loads and distributed generation. SOE (Switch Opening and Exchange) solution technique. |
| ***Metaheuristics*** | [24] | 1990 | Chiang y Jean-Jumeau | Simulated annealing algorithm. |
| [25] | 2004 | Guimaraes et. al | Tabu search algorithm. |
| [26] | 2008 | Chang | Ant colony algorithm. |
| [27] | 2020 | Reza et al. | Cuckoo Search Algorithm incorporating stochasticity of demand and distributed generation. |
| ***Artificial intelligence*** | [28] | 1993 | Kim et. al | Use of artificial neural networks for the classification of load level and zone reconfiguration. |
| [3] | 1999 | Kangan y Barioni | Fuzzy logic model. |
| [29] | 2006 | Salazar et. al | Artificial neural network |
| [30] | 2022 | Roshni et a. | Reinforcement learning (RL). |

As it is evident from Table 1 and the literature review, while the minimization of line losses has been a major focus of researchers in the reconfiguration problem, other highly popular objective functions include power quality and the maximization of distributed generation [7]. Currently, the most popular area for research related to the reconfiguration problem is the operation of smart distribution systems with the presence of distributed generation, energy storage systems, electric vehicles, among others [7]; however, research on mathematical formulation continue to be of importance due to the necessity of faster models to be successfully implemented in real EMSs [5].

3. Mathematical model

3.1. General mathematical model

By applying the law of conservation of energy (i.e., Kirchhoff's current law) and Ohm's law, it is possible to define a mathematical model that allows estimating the power flow in the distribution network. The following non-linear model expresses the netwrok reconfiguration problema as MINLP that, in principle can be solved by an optimizer such as Gurobi:

|  |  |
| --- | --- |
|  | ( 1) |
|  | ( 2) |
|  | ( 3) |
|  | ( 4) |
|  | ( 5) |
|  | ( 6) |
|  | ( 7) |
|  | ( 8) |
|  | ( 9) |

From the definition of electrical losses in a conductor, , it is possible to define the objective function according to equation ( 1).

The constraints are then given by the principle of energy conservation:

|  |  |
| --- | --- |
|  | ( 10) |

Equation (2) expresses the law of conservation of energy (i.e., Kirchhoff's current law) as a balance between the powers entering and leaving a node. Referring to **Figure 1**, it is possible to separate the power balance into two parts: an active power balance and a reactive power balance. The active power balance for a node I, is expressed as (*2*).



**Figure 1**. Active and reactive power balance at node i.

Similarly, the reactive power balance at node i is expressed according to equation (*3*).

The voltage drop across branch k-i is governed by Ohm's law. Referring to **Figure 1**, equation (*11*) can be written, where the variables highlighted in bold denote phasorial quantities.

|  |  |
| --- | --- |
|  | ( 11) |

Following the procedure devised by [31], it is possible to transform Equation (*11*) into a simpler form (*12*), in which there are no longer phasorial quantities.

|  |  |
| --- | --- |
|  | ( 12) |

However, as a trade-off, ( *12*) represents the first nonlinear constraint. In (*12*) , the value refers to the impedance of the transmission line, given by (*13*).

|  |  |
| --- | --- |
|  | ( 13) |

In accordance with [19], constraint (*12*) can be modified to include the effect of the circuit breaker present in branch i-k, as shown in Equation (*4*); The value of is given by (*5*) [19].

The relationship among voltages, powers, and currents can be easily expressed by the well-known power triangle [32] as expressed by constraint (*6*).

The constraints expressing the safety and reliability conditions of the distribution network, ensuring that voltages and currents are within acceptable limits, are provided by equations (*7*) and (*8*). Finally, the radiality conditions that ensure a radial topology in the distribution system are expressed in constraint (*9*) [11].

Thus, the exact mathematical model for the DPSR is composed by objective function ( 1) and constraints (*2*), (*3*), (*4*), (*5*), (*6*), (*7*), (*8*) and (*9*); as it can be seen, this model is a mixed integer non-linear non-convex optimization problem [3], [7].

3.2. Linearization of term

By considering the problem variables as and instead of  and , the only non-linear constraint is constraint (6). It is well known that, in per unit (p.u.) base system, the voltage magnitude is very close to 1 p.u. at all buses in the distribution network. Therefore, a simplification can be proposed as shown in constraint (14) [33]. This approach is simpler that the one commonly used to linearize the power triangle constraint [2], [3], and as it will be shown in the results section, it is very accurate and allow to reduce the computational effort.

|  |  |
| --- | --- |
|  | ( 14) |

Besides, constraint (14) can be expressed through convex relaxation in the form of (15). This transformation renders the feasible space convex, albeit slightly larger than the original space. Consequently, the optimal value found may be smaller than the true optimal value. However, it has been verified that constraint (15*)* is binding in optimality [34], [35]. Constraint (15*)* can be solved using the Branch and Bound algorithm [34], and is readily accepted by commercial software such as ***Gurobi***.

|  |  |
| --- | --- |
|  | (15) |

Other possibility is using the group of constraints (16) and (17). As it will be shown in the results section, (16*)* and (17) is a good compromise between computational time and solution accuracy.

|  |  |
| --- | --- |
|  | (16)  ( 17) |

Finally, the radiality constraint expressed in (*9*) can be reformulated as shown in (18). The idea behind this set of constrains is that a bus bar can only be fed by one conductor to ensure the radiality topology formation. With this modification, a single constraint ( *9*) is replaced by as many constraints (18*)* as there are non-substation nodes in the network. This has the potential to enhance solution speed, as certain variables may be assigned specific values during the pre-solve step since, in some networks, certain nodes only have one connection The proposed set of constraints also functions effectively for bus bars with zero demand, preventing the generation of non-connected or non-radial topologies.

|  |  |
| --- | --- |
|  | (18) |
|  |  |

4. Numerical Results

In this section the five instances for testing used to evaluate the models described in Section 3 are detailed below. Subsequently, the numerical results obtained are presented, followed by a brief discussion of the findings.

4.1. Instances’ description

The previously mentioned improvement strategies were evaluated using five case studies sourced from the Laboratory of Electric Power Systems Planning at the Department of Electrical Engineering of the São Paulo State University [36]. Each case study is distinguished by the number of branches (connections between nodes) and corresponding information about the loads at each bu. Table 2 includes data on the number of switches, substation node, and base voltages and powers for each considered system. Typically, the literature utilizes small instances [30] (13 and 33 nodes), medium instances [2] (84, 119, 136 and 208 nodes) and big instances [37] (417 barras).

**Table 2**. Instances’data

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| # of nodes | # of switches | Ref. node | Base Voltage (kV) | Base apparent power (MVA) |
| 14 | 16 | 14 | 23 | 100 |
| 33 | 74 | 1 | 12.66 | 10 |
| 84 | 192 | 84 | 11.4 | 10 |
| 136 | 312 | 1 | 13.8 | 100 |
| 417 | 473 | 1 | 10 | 100 |

The metrics to be studied in this section are defined.

* Active power losses: These refer to the power losses due to Joule effect and are estimated using the objective function(1).
* Computation time: This corresponds to the time taken by the code to load the data, build the model in the Gurobi optimizer, and solve the system. The total time refers to the sum of these three times.

In **Table 3** the overall results obtained for the different models studied are presented, applied to the test systems with 14, 33, 84, 133, and 417 buses. An AMD Ryzen 5 3550H processor with a clock speed of 2100 MHz and 4 cores was used as the computing equipment. The simulation time limit was set at 3600 seconds. For a clearer understanding of each model, a brief description is provided below:

* *Base model*

The base model consists of constraints (*2*), (*3*), (*4*), (*5*), (*6*), (*7*), (*8*) and (*9*).

* *Model with modified radiality constraints*

The base model can be modified by including the modified radiality constraints. In this case, the incumbent constraints are (*2*), (*3*), (*4*), (*5*), (*6*), (*7*), (*8*) and (18)

* *Model with a simplification of the term*

In this case, constraint (16) is included instead of constraint (14); the model then consists of constraints (*2*), (*3*), (*4*), (*5*), (15), (*7*), (*8*) and (18)

* *Model with a double power triangle constraint.*

In this case, the set of constraints (17) is included instead of constraint (16); the model then consists of constraints (*2*), (*3*), (*4*), (*5*), (15), (16), (*7*), (*8*) and (18)

4.2. Discussion of results

As seen in **Table 3** , a significant reduction in computation time is achieved by replacing the radiality constraint (13) with the set of constraints (18). This improvement is even more evident in test systems with higher number of nodes (84 and 136). A further reduction in computation time is accomplished by simplifying the constraint (14), assuming that the voltage variation at the nodes is negligible and performing the convex relaxation (17). Moreover, the model with this simplification is one who is closer to an optimality gap o 0.0%.

Due to the simplification made to constraint (17*)*, this model is an approximate model; in order to overcome the limitations of the simplification (17*)*, a Gauss-Seidel [38] power

**Table 3**. Results of the three considered models.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Ins** | **Base model** | | | | **Model with radiality modified** | | | **Model with a simplification**  **of the term** | | | |  | **Double power triangle**  **constraint.** | | |
| **TT** | **PPA** | **GO** | | **TT** | **PPA** | **GO** | **TT** | **PPA** | **GO** | **OF** | **%ER** | **TT** | **PPA** | **GO** |
| 14 | 0.509 | 605.9 | 0 | | 0.8642 | 605.9 | 0 | 0.4471 | 605.7 | 0 | 577.7 | 4.62 | 0.4991 | 605.9 | 0 |
| 33 | 1.735 | 139.4 | 0 | | 1.1662 | 139.4 | 0 | 1.0551 | 139,2 | 0 | 131.8 | 5.31 | 1.0024 | 139.4 | 0 |
| 84 | 8.0939 | 469.3 | 0 | | 3.0809 | 469.3 | 0 | 2.7497 | 468,6 | 0 | 447.4 | 4.52 | 3.7231 | 469.3 | 0 |
| 133 | 3601.85\*\* | NSF | NSF | | 82.785 | 279.6 | 0 | 8.0224 | 287.9 | 0 | 265.8 | 6.7 | 32.702 | 279.6 | 0 |
| 417 | 3606.13\*\* | NSF | NSF | | 3606.6\*\* | 1653\*\*\* | 68.5 | 3606.3\*\* | 581.5\*\*\* | 2.55 | 565.0 | 2.8 | 3606.8 | 582.5 | 3.0 |
| Ins: Instancia  TT: Total time (s)  PPA: Active power losses (kW) GO: Solver gap (%) | | | | OF: Objective function (kW)  %ER: Percental difference among objective function and losses computed by Gauss Seidel power flow analysis.  \*\*: Wal time reached.  \*\*\*: Unable to find the optimum value.  NSF: Unable to find a feasible solution | | | | | | | | | | | |

**Table 4.** Results from the literature.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Instance** | **Lit. Solution (kW)** | **Solution time (s)** | **CPU** | **Reference** |
| 33 | 139.54 | 0.46 | Intel i7-8850H | [2] |
| 84 | 469.87 | 2.58 | Intel i7-8850H | [2] |
| 133 | 280.19 | 7.07 | Intel i7-8850H | [2] |
| 417 | 581.57 | 171425 | Inter i7PC @1.87 GHz | [3] |

flow analysis was conducted on the solutions found. In **Table 3** the percentage difference between the objective function and the losses computed using Gauss-Seidel power flow analysis is presented. As is evident, this model tends to underestimate power losses because it assumes constant voltage at every node for the power triangle constraint. As can be seen in the annex table A.4, the solution obtained through this approximate method differs slightly from the configuration found with the base model and the model with modified radial conditions. For the instance with 133 nodes, the difference between the power loss results obtained by this model and those reported in the literature is 3%, with an optimality gap of 0%. For the instance with 417 nodes, the difference between the power loss results obtained by this model and those reported in the literature is 0.03%, with an optimality gap of 2.55%.

Finally, the set of constraints (16) and (17), allows solving the optimization problem within an intermediate computation time between those achieved with constraint (14) and constraint (16), making it an exact model and a good compromise between precision and computational effort. However, this model is also unable to solve the reconfiguration problem for the 417-node instance within the 3600s time limit, with an optimality gap of 3%, significantly better than the base model or the model with modified radiality constraints.

In order to validate the proposed models in Section 3, a comparison of the obtained solutions with those reported in the literature is conducted. **Table 4** shows the values of reported and obtained active power losses for the test systems with 33, 84, 136 nodes, and 417 nodes. As it can be seen, there are significant differences only for the model that assumes a constant voltage of 1 p.u for every node in the power triangle constraint; even though, this difference is smaller when the power losses are calculated using the Gauss-Seidel power flow analysis on the topology found by the model (PPA column on **Table 3**).

5. Conclusions

As evident from the results presented in Section 4, the computation time of the mixed-integer linear model significantly improves by replacing the radiality constraint proposed by [11], with a larger set of constraints, each involving fewer variables (18). Another substantial improvement is achieved by linearizing the apparent power in equation (16); however, this procedure makes this model to underestimate the power losses and, in some cases to achieve a slightly different solution than the one gotten by using constraint (16). To overcome this issue, it is highly recommended to perform a power flow analysis on the solutions found with this model. While the best computation times for the models considered in this study are achieved by following the approach of [33], the set of constraints (16), (17) proposed in this work represent a good compromise between computation time and solution accuracy, without the necessity of performing a subsequent power flow analysis.

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**Appendix A**

1. **Instances’ solutions**

The solution for each case study, in terms of branches, is presented below for each proposed model and instance.

A.1 Instancia de 14 barras

Tabla 1. 14 nodes instance solution tree

|  |  |
| --- | --- |
| **Modelo** | **Optimal Tree** |
| Base model | (14-13), (14-9), (14-4), (13-12), (13-11), (12-6), (11-10), (9-8), (8-5), (4-3), (4-2), (3-7), (2-1) |
| Model with radiality reformulated | (14-13), (14-9), (14-4), (13-12), (13-11), (12-6), (11-10), (9-8), (8-5), (4-3), (4-2), (3-7), (2-1) |
| Linearization of term | (14-13), (14-9), (14-4), (13-12), (13-11), (12-6), (11-10), (9-8), (8-5), (4-3), (4-2), (3-7), (2-1) |
| Double constraint for power triangle | (14-13), (14-9), (14-4), (13-12), (13-11), (12-6), (11-10), (9-8), (8-5), (4-3), (4-2), (3-7), (2-1) |

14

2

9

12

6

8

5

1

10

11

13

7

4

3

2

9

14

12

6

8

5

1

10

11

13

7

4

3

(a)

(b)

Figura 1. (a) Initial configuration y (b) Final configuration

A.2 Instancia de 33 barras

Tabla 2. 33 nodes instance

|  |  |
| --- | --- |
| **Model** | **Optimal tree** |
| Base model | (1-2), (2-3), (2-19), (3-4), (3-23), (4-5), (5-6), (6-7), (6-26), (8-9), (9-15), (11-10), (12-11), (12-13), (13-14), (15-16), (16-17), (17-18), (18-33), (19-20), (20-21), (21-22), (21-8), (22-12), (23-24), (24-25), (26-27), (27-28), (28-29), (29-30), (30-31), (31-32) |
| Model with radiality reformulated | (1-2), (2-3), (2-19), (3-4), (3-23), (4-5), (5-6), (6-7), (6-26), (8-9), (9-15), (11-10), (12-11), (12-13), (13-14), (15-16), (16-17), (17-18), (18-33), (19-20), (20-21), (21-22), (21-8), (22-12), (23-24), (24-25), (26-27), (27-28), (28-29), (29-30), (30-31), (31-32) |
| Linearization of term | (1-2), (2-3), (2-19), (3-4), (3-23), (4-5), (5-6), (6-7), (6-26), (8-9), (9-15), (11-10), (12-11), (12-13), (13-14), (15-16), (16-17), (17-18), (18-33), (19-20), (20-21), (21-22), (21-8), (22-12), (23-24), (24-25), (26-27), (27-28), (28-29), (29-30), (30-31), (31-32) |
| Double constraint for power triangle | (1-2), (2-3), (2-19), (3-4), (3-23), (4-5), (5-6), (6-7), (6-26), (8-9), (9-15), (11-10), (12-11), (12-13), (13-14), (15-16), (16-17), (17-18), (18-33), (19-20), (20-21), (21-22), (21-8), (22-12), (23-24), (24-25), (26-27), (27-28), (28-29), (29-30), (30-31), (31-32) |

A.3 84 nodes instance.

Tabla 3. 84 nodes instances

|  |  |
| --- | --- |
| **Model** | **Optimal tree** |
| Base model | (1-2), (84-1), (84-11), (84-15), (84-25), (84-30), (84-43), (84-47), (84-56), (84-65), (84-73), (84-77), (2-3), (3-4), (4-5), (5-6), (5-55), (7-8), (7-9), (7-10), (11-12), (12-14), (12-72), (15-16), (16-17), (17-18), (18-19), (19-20), (20-21), (20-83), (21-22), (21-23), (23-24), (25-26), (26-27), (27-28), (28-29), (29-39), (30-31), (31-32), (32-33), (34-35), (35-36), (36-37), (37-38), (38-41), (39-40), (40-42), (43-44), (44-45), (45-46), (46-34), (47-48), (48-49), (49-50), (50-51), (51-52), (52-53), (53-54), (53-64), (56-57), (57-58), (58-59), (59-60), (60-61), (60-7), (63-62), (64-63), (65-66), (66-67), (67-68), (68-69), (69-70), (70-71), (73-74), (74-75), (75-76), (76-13), (77-78), (78-79), (79-80), (80-81), (81-82) |
| Model with radiality reformulated | (1-2), (84-1), (84-11), (84-15), (84-25), (84-30), (84-43), (84-47), (84-56), (84-65), (84-73), (84-77), (2-3), (3-4), (4-5), (5-6), (5-55), (7-8), (7-9), (7-10), (11-12), (12-14), (12-72), (15-16), (16-17), (17-18), (18-19), (19-20), (20-21), (20-83), (21-22), (21-23), (23-24), (25-26), (26-27), (27-28), (28-29), (29-39), (30-31), (31-32), (32-33), (34-35), (35-36), (36-37), (37-38), (38-41), (39-40), (40-42), (43-44), (44-45), (45-46), (46-34), (47-48), (48-49), (49-50), (50-51), (51-52), (52-53), (53-54), (53-64), (56-57), (57-58), (58-59), (59-60), (60-61), (60-7), (63-62), (64-63), (65-66), (66-67), (67-68), (68-69), (69-70), (70-71), (73-74), (74-75), (75-76), (76-13), (77-78), (78-79), (79-80), (80-81), (81-82) |
| Linearization of term | (1-2), (84-1), (84-11), (84-15), (84-25), (84-30), (84-43), (84-47), (84-56), (84-65), (84-73), (84-77), (2-3), (3-4), (4-5), (5-6), (5-55), (7-8), (7-9), (7-10), (11-12), (12-14), (12-72), (15-16), (16-17), (17-18), (18-19), (19-20), (20-21), (20-83), (21-22), (21-23), (23-24), (25-26), (26-27), (27-28), (28-29), (29-39), (30-31), (31-32), (32-33), (34-35), (35-36), (36-37), (37-38), (38-41), (39-40), (40-42), (43-44), (44-45), (45-46), (46-34), (47-48), (48-49), (49-50), (50-51), (51-52), (52-53), (53-54), (53-64), (56-57), (57-58), (58-59), (59-60), (60-61), (60-7), (63-62), (64-63), (65-66), (66-67), (67-68), (68-69), (69-70), (70-71), (73-74), (74-75), (75-76), (76-13), (77-78), (78-79), (79-80), (80-81), (81-82) |
| Double constraint for power triangle | (1-2), (84-1), (84-11), (84-15), (84-25), (84-30), (84-43), (84-47), (84-56), (84-65), (84-73), (84-77), (2-3), (3-4), (4-5), (5-6), (5-55), (7-8), (7-9), (7-10), (11-12), (12-14), (12-72), (15-16), (16-17), (17-18), (18-19), (19-20), (20-21), (20-83), (21-22), (21-23), (23-24), (25-26), (26-27), (27-28), (28-29), (29-39), (30-31), (31-32), (32-33), (34-35), (35-36), (36-37), (37-38), (38-41), (39-40), (40-42), (43-44), (44-45), (45-46), (46-34), (47-48), (48-49), (49-50), (50-51), (51-52), (52-53), (53-54), (53-64), (56-57), (57-58), (58-59), (59-60), (60-61), (60-7), (63-62), (64-63), (65-66), (66-67), (67-68), (68-69), (69-70), (70-71), (73-74), (74-75), (75-76), (76-13), (77-78), (78-79), (79-80), (80-81), (81-82) |

A.4 133 nodes instance

Tabla 4. 133 nodes instances

| **Modelo** | **Árbol óptimo** |
| --- | --- |
| Base model | No feasible solution found |
| Model with radiality reformulated | (136-1), (136-17), (136-39), (136-63), (136-75), (136-85), (136-99), (136-121), (1-2), (2-3), (3-4), (4-5), (5-6), (6-8), (8-9), (8-10), (10-11), (10-12), (10-13), (13-14), (13-15), (15-16), (17-18), (18-19), (19-20), (20-21), (20-22), (22-23), (22-24), (24-25), (25-26), (25-51), (26-27), (27-28), (28-29), (28-31), (29-30), (31-32), (32-33), (33-34), (35-36), (36-37), (38-35), (39-40), (40-41), (40-42), (42-43), (43-44), (43-45), (45-46), (46-47), (47-48), (47-62), (47-110), (48-49), (49-50), (51-52), (52-53), (52-56), (53-54), (56-57), (57-58), (58-59), (59-60), (60-61), (62-120), (63-64), (64-65), (65-66), (66-67), (67-68), (68-69), (68-70), (70-71), (70-73), (71-72), (73-74), (73-7), (75-76), (76-77), (76-126), (77-78), (78-79), (79-80), (80-81), (81-82), (81-83), (83-84), (85-86), (86-87), (86-88), (88-89), (90-91), (91-92), (92-93), (93-94), (93-97), (94-95), (97-98), (98-55), (98-135), (99-100), (100-101), (101-102), (101-103), (103-104), (103-90), (104-105), (107-106), (107-108), (108-109), (108-114), (109-116), (110-107), (110-111), (111-112), (112-113), (114-115), (116-117), (119-118), (120-119), (120-96), (121-122), (122-123), (123-124), (123-125), (125-127), (127-128), (127-129), (129-130), (130-131), (131-132), (132-133), (133-134), (135-38) |
| Linearization of term | (136-1), (136-17), (136-39), (136-63), (136-75), (136-85), (136-99), (136-121), (1-2), (2-3), (3-4), (4-5), (5-6), (6-8), (8-10), (10-11), (10-12), (10-13), (13-14), (13-15), (15-16), (17-18), (18-19), (19-20), (20-21), (20-22), (22-23), (22-24), (24-25), (24-9), (25-26), (25-51), (26-27), (27-28), (28-29), (28-31), (29-30), (31-32), (31-35), (32-33), (33-34), (35-36), (36-37), (39-40), (40-41), (40-42), (42-43), (43-44), (43-45), (45-46), (46-47), (47-48), (47-62), (47-110), (48-49), (49-50), (50-96), (51-52), (52-56), (54-53), (55-54), (56-57), (57-58), (58-59), (59-60), (60-61), (62-120), (63-64), (64-65), (65-66), (66-67), (67-68), (68-69), (68-70), (70-71), (70-73), (71-72), (73-74), (73-7), (75-76), (76-77), (77-78), (78-79), (79-80), (80-81), (81-82), (81-83), (85-86), (86-87), (86-88), (88-89), (90-91), (91-92), (92-93), (93-94), (93-97), (96-95), (97-98), (98-55), (99-100), (100-101), (101-102), (101-103), (103-104), (103-90), (104-105), (104-118), (105-106), (107-108), (108-109), (108-114), (109-116), (110-107), (110-111), (111-112), (112-113), (114-115), (116-117), (120-119), (121-122), (122-123), (123-124), (123-125), (125-126), (125-127), (127-128), (127-129), (129-130), (130-131), (131-132), (132-133), (133-134), (134-135), (135-38), (135-84) |
| Double constraint for power triangle | (136-1), (136-17), (136-39), (136-63), (136-75), (136-85), (136-99), (136-121), (1-2), (2-3), (3-4), (4-5), (5-6), (6-8), (8-9), (8-10), (10-11), (10-12), (10-13), (13-14), (13-15), (15-16), (17-18), (18-19), (19-20), (20-21), (20-22), (22-23), (22-24), (24-25), (25-26), (25-51), (26-27), (27-28), (28-29), (28-31), (29-30), (31-32), (32-33), (33-34), (35-36), (36-37), (38-35), (39-40), (40-41), (40-42), (42-43), (43-44), (43-45), (45-46), (46-47), (47-48), (47-62), (47-110), (48-49), (49-50), (51-52), (52-53), (52-56), (53-54), (56-57), (57-58), (58-59), (59-60), (60-61), (62-120), (63-64), (64-65), (65-66), (66-67), (67-68), (68-69), (68-70), (70-71), (70-73), (71-72), (73-74), (73-7), (75-76), (76-77), (76-126), (77-78), (78-79), (79-80), (80-81), (81-82), (81-83), (83-84), (85-86), (86-87), (86-88), (88-89), (90-91), (91-92), (92-93), (93-94), (93-97), (94-95), (97-98), (98-55), (98-135), (99-100), (100-101), (101-102), (101-103), (103-104), (103-90), (104-105), (107-106), (107-108), (108-109), (108-114), (109-116), (110-107), (110-111), (111-112), (112-113), (114-115), (116-117), (119-118), (120-119), (120-96), (121-122), (122-123), (123-124), (123-125), (125-127), (127-128), (127-129), (129-130), (130-131), (131-132), (132-133), (133-134), (135-38) |

A.4 Instancia de 417 barras

Tabla 5. 417 nodes instances

|  |  |
| --- | --- |
| **Modelo** | **Árbol óptimo** |
| **Base model** | No feasible solution found |
| **Model with radiality reformulated** | (68-93), (68-67), (89-88), (67-89), (67-64), (225-224), (224-222), (21-47), (21-50), (21-49), (21-57), (72-21), (72-74), (66-252), (3-20), (3-9), (20-44), (20-33), (222-221), (74-75), (382-70), (382-66), (51-52), (221-344), (52-56), (220-223), (75-81), (81-76), (59-77), (56-27), (1-273), (1-274), (1-123), (1-92), (1-351), (1-350), (1-211), (1-215), (1-190), (1-373), (1-362), (1-364), (85-68), (27-31), (274-275), (31-40), (275-276), (73-72), (73-80), (60-61), (58-60), (209-208), (209-210), (80-82), (65-69), (71-65), (47-34), (276-264), (77-73), (69-59), (54-51), (264-263), (24-54), (263-262), (79-78), (83-79), (63-71), (61-63), (29-24), (262-261), (34-29), (261-259), (259-257), (90-83), (62-58), (257-254), (257-258), (64-62), (44-32), (372-239), (239-240), (239-243), (123-120), (265-339), (265-367), (120-94), (94-113), (94-111), (94-119), (94-98), (35-25), (369-347), (369-370), (38-35), (370-348), (370-383), (370-90), (92-2), (92-87), (41-38), (112-115), (2-17), (2-14), (2-13), (43-41), (383-382), (113-114), (48-43), (17-18), (50-48), (50-46), (208-277), (208-236), (277-278), (277-279), (106-95), (95-112), (6-7), (7-3), (33-55), (33-22), (84-104), (84-109), (84-107), (279-280), (111-106), (18-5), (5-6), (28-36), (36-45), (280-281), (119-121), (119-122), (119-200), (49-39), (235-234), (14-16), (55-23), (23-28), (282-283), (234-282), (234-233), (122-118), (236-235), (16-19), (19-8), (104-105), (232-293), (207-349), (13-4), (42-37), (380-229), (229-228), (109-108), (4-12), (12-15), (46-42), (228-371), (15-11), (231-232), (11-86), (11-10), (103-110), (107-103), (86-91), (53-26), (22-53), (304-378), (304-303), (304-311), (306-304), (97-117), (116-97), (26-30), (378-377), (99-84), (101-100), (96-101), (309-314), (307-309), (307-312), (100-102), (314-317), (314-318), (317-313), (102-99), (187-167) (167-175), (318-319), (138-194), (138-149), (117-96), (175-181), (301-305), (301-302), (305-310), (305-308), (155-138), (151-155), (151-152), (181-160), (181-178), (310-315), (310-306), (310-320), (124-126), (124-158), (98-116), (160-180), (180-177), (180-174), (351-379), (129-141), (129-154), (129-151), (346-129), (350-203), (162-169), (203-204), (203-206), (153-132), (141-153), (141-130), (183-162), (379-380), (142-147), (182-183), (182-186), (204-205), (136-142), (243-384), (243-355), (243-356), (243-246), (243-249), (132-136), (244-85), (206-207), (163-171), (352-414), (352-212), (352-353), (414-385), (414-401), (184-163), (184-187), (125-128), (401-386), (401-402), (375-202), (127-124), (128-127), (128-125), (131-157), (157-133), (402-387), (402-403), (130-131), (211-209), (200-201), (403-388), (403-404), (133-140), (202-375), (404-400), (404-405), (404-407), (368-345), (368-343), (345-346), (345-369), (373-352), (199-192), (199-197), (154-156), (154-134), (197-191), (137-144), (144-148), (353-213), (353-214), (159-199), (159-161), (159-182), (159-184), (148-150), (156-137), (405-389), (405-406), (190-159), (190-189), (215-216), (161-172), (161-185), (150-143), (406-390), (406-391), (216-217), (216-218), (172-188), (188-168), (134-139), (218-219), (168-176), (407-392), (407-408), (219-220), (176-179), (408-409), (408-410), (223-237), (185-164), (409-393), (409-394), (237-238), (237-372), (164-165), (145-146), (165-173), (152-135), (135-145), (135-195), (410-415), (410-411), (411-395), (411-412), (355-354), (354-242), (354-241), (186-166), (412-399), (412-398), (412-413), (166-170), (189-198), (189-193), (198-196), (356-357), (413-397), (413-396), (357-244), (357-245), (290-292), (290-291), (289-290), (246-247), (247-248), (266-265), (266-324), (288-289), (267-266), (300-288), (300-286), (300-301), (300-307), (249-358), (268-267), (268-327), (268-297), (358-359), (269-268), (359-250), (365-269), (365-366), (285-300), (366-270), (366-271), (360-251), (363-365), (363-381), (253-360), (362-363), (362-272), (254-253), (254-361), (254-255), (361-256), (296-260), (296-299), (296-298), (260-287), (312-316), (364-284), (364-285), (324-326), (324-325), (334-336), (334-335), (336-323), (327-328), (338-337), (322-329), (233-376), (297-296), (339-338), (321-322), (376-230), (299-321), (230-231), (367-340), (340-341), (329-330), (329-333), (371-374), (294-295), (341-342), (341-368), (330-331), (374-227), (374-226), (292-294), (331-332), (226-225), (333-334) |
| **Linearization of term** | (384-68), (68-93), (68-67), (89-88), (67-89), (67-64), (225-224), (224-222), (21-47), (21-50), (72-21), (72-74), (72-76), (3-20), (20-51), (20-44), (222-221), (74-75), (70-59), (382-70), (51-52), (51-54), (221-344), (76-81), (52-56), (220-223), (59-69), (56-27), (1-273), (1-274), (1-123), (1-92), (1-351), (1-350), (1-375), (1-211), (1-215), (1-190), (1-373), (1-362), (1-364), (274-275), (275-276), (73-72), (73-80), (60-58), (40-31), (209-208), (209-210), (80-82), (65-71), (71-63), (47-40), (276-264), (77-73), (69-65), (54-24), (264-263), (264-265), (78-77), (24-29), (263-262), (79-78), (83-79), (63-61), (61-60), (29-34), (262-261), (261-259), (90-83), (257-254), (257-258), (64-62), (44-25), (44-32), (372-239), (239-240), (239-243), (123-120), (25-35), (265-339), (265-367), (120-94), (94-113), (94-111), (94-119), (94-98), (35-38), (369-347), (369-370), (38-41), (370-348), (370-90), (114-115), (115-112), (92-2), (92-87), (2-17), (2-14), (2-13), (383-382), (251-383), (251-359), (251-252), (113-114), (48-43), (17-18), (50-48), (50-46), (208-277), (208-236), (277-278), (277-279), (106-95), (6-7), (33-55), (33-22), (84-33), (84-104), (84-109), (84-110), (84-107), (279-280), (111-106), (18-5), (5-6), (28-36), (36-45), (280-281), (45-49), (119-121), (119-122), (119-200), (49-39), (235-234), (14-16), (55-23), (23-28), (282-283), (118-84), (8-9), (9-3), (234-282), (122-118), (236-235), (16-19), (19-8), (104-105), (32-37), (232-231), (232-293), (349-232), (37-42), (207-349), (13-4), (380-229), (229-228), (109-108), (4-12), (12-15), (228-371), (15-11), (231-230), (110-103), (11-86), (86-91), (53-26), (22-53), (304-378), (97-117), (116-97), (26-30), (378-377), (101-100), (96-101), (88-10), (309-314), (309-313), (307-309), (307-312), (100-102), (57-147), (57-149), (57-148), (314-318), (314-310), (317-319), (102-99), (187-167), (167-175), (318-315), (138-194), (117-96), (175-181), (301-305), (301-302), (305-304), (151-155), (151-152), (181-178), (310-306), (310-320), (315-311), (124-126), (124-127), (98-116), (160-129), (180-160), (180-177), (351-379), (129-141), (129-154), (129-151), (350-203), (147-140), (162-169), (203-204), (203-206), (153-132), (141-153), (141-130), (183-162), (379-380), (182-183), (182-186), (252-66), (204-205), (136-142), (243-384), (243-355), (243-356), (243-246), (243-249), (132-136), (244-85), (206-207), (163-171), (352-414), (352-212), (352-353), (414-385), (414-401), (149-146), (149-138), (184-163), (184-187), (125-57), (401-386), (401-402), (375-202), (127-128), (128-125), (131-157), (157-133), (402-387), (402-403), (130-131), (211-209), (200-201), (200-158), (403-388), (403-404), (158-124), (404-400), (404-405), (404-407), (368-345), (368-343), (345-346), (345-369), (373-352), (199-192), (199-197), (154-156), (154-134), (197-191), (137-144), (353-213), (353-214), (159-199), (159-161), (159-182), (159-184), (148-150), (156-137), (405-389), (405-406), (190-159), (190-189), (215-216), (161-172), (161-185), (150-143), (406-390), (406-391), (216-217), (216-218), (172-188), (188-168), (134-139), (218-219), (168-176), (407-392), (407-408), (219-220), (176-179), (179-173), (408-409), (408-410), (223-237), (185-164), (409-393), (409-394), (237-238), (237-372), (164-165), (152-135), (135-145), (135-195), (410-415), (410-411), (411-395), (411-412), (355-354), (354-242), (354-241), (186-166), (412-399), (412-398), (412-413), (166-170), (189-198), (189-193), (170-174), (198-196), (356-357), (174-180), (413-397), (413-396), (357-244), (357-245), (290-291), (289-290), (246-247), (247-248), (266-324), (288-289), (267-266), (300-288), (300-286), (300-301), (300-307), (268-267), (268-327), (268-297), (268-292), (269-268), (359-358), (359-250), (365-269), (365-366), (285-300), (313-317), (366-270), (366-271), (302-303), (360-251), (363-365), (363-381), (253-360), (362-363), (362-272), (254-253), (254-361), (254-255), (361-256), (316-308), (296-260), (296-299), (296-298), (260-287), (312-316), (364-284), (364-285), (324-326), (324-325), (324-323), (334-335), (283-257), (323-336), (327-328), (338-337), (322-329), (297-296), (339-338), (321-322), (376-233), (299-321), (230-376), (367-340), (340-341), (329-330), (329-333), (371-374), (294-295), (341-342), (341-368), (330-331), (374-227), (374-226), (292-294), (331-332), (226-225), (333-334) |
| **Double constraint for power triangle** | (384-68), (384-66), (68-93), (68-67), (89-88), (67-89), (67-64), (225-224), (224-222), (21-47), (21-50), (72-21), (72-74), (72-76), (3-20), (20-51), (20-44), (222-221), (74-75), (70-59), (382-70), (51-52), (51-54), (221-344), (76-81), (52-56), (220-223), (59-69), (56-27), (1-273), (1-274), (1-123), (1-92), (1-351), (1-350), (1-375), (1-211), (1-215), (1-190), (1-373), (1-362), (1-364), (274-275), (275-276), (73-72), (73-80), (60-58), (40-31), (209-208), (209-210), (80-82), (65-71), (71-63), (47-40), (276-264), (77-73), (69-65), (54-24), (264-263), (264-265), (78-77), (24-29), (263-262), (79-78), (83-79), (63-61), (61-60), (29-34), (262-261), (90-83), (257-259), (257-254), (257-258), (64-62), (44-25), (44-32), (372-239), (239-240), (239-243), (123-120), (25-35), (265-339), (265-367), (120-94), (94-113), (94-111), (94-119), (94-98), (35-38), (369-347), (369-370), (38-41), (370-348), (370-90), (114-115), (115-112), (92-2), (92-87), (2-17), (2-14), (2-13), (383-382), (251-383), (251-359), (251-252), (113-114), (48-43), (17-18), (50-48), (50-46), (208-277), (208-236), (277-278), (277-279), (106-95), (6-7), (33-55), (33-22), (84-33), (84-104), (84-109), (84-110), (84-107), (279-280), (111-106), (18-5), (5-6), (28-36), (36-45), (280-281), (45-49), (119-121), (119-122), (119-200), (235-234), (14-16), (55-23), (23-28), (282-283), (118-84), (8-9), (9-3), (234-282), (122-118), (236-235), (16-19), (19-8), (104-105), (32-37), (232-231), (232-293), (349-232), (37-42), (207-349), (13-4), (380-229), (229-228), (109-108), (4-12), (12-15), (228-371), (15-11), (231-230), (110-103), (11-86), (30-39), (86-91), (53-26), (22-53), (304-378), (97-117), (116-97), (26-30), (378-377), (101-100), (96-101), (88-10), (309-314), (309-313), (307-309), (307-312), (100-102), (57-149), (57-148), (314-318), (314-310), (317-319), (102-99), (187-167), (167-175), (318-315), (138-194), (117-96), (175-181), (301-305), (301-302), (305-304), (305-308), (151-155), (151-152), (181-178), (310-306), (310-320), (315-311), (124-126), (124-127), (98-116), (160-129), (160-179), (180-160), (180-177), (351-379), (129-141), (129-154), (129-151), (350-203), (162-169), (203-204), (203-206), (153-132), (141-153), (141-130), (183-162), (379-380), (142-147), (182-183), (182-186), (204-205), (136-142), (243-384), (243-355), (243-356), (243-246), (243-249), (132-136), (244-85), (206-207), (163-171), (352-414), (352-212), (352-353), (414-385), (414-401), (149-146), (149-138), (184-163), (184-187), (125-57), (401-386), (401-402), (375-202), (127-128), (128-125), (131-157), (157-133), (402-387), (402-403), (130-131), (211-209), (200-201), (200-158), (403-388), (403-404), (133-140), (158-124), (404-400), (404-405), (404-407), (368-345), (368-343), (345-346), (345-369), (373-352), (199-192), (199-197), (154-156), (154-134), (197-191), (137-144), (353-213), (353-214), (159-199), (159-161), (159-182), (159-184), (148-150), (156-137), (405-389), (405-406), (190-159), (190-189), (215-216), (161-172), (161-185), (150-143), (406-390), (406-391), (216-217), (216-218), (172-188), (188-168), (134-139), (218-219), (168-176), (407-392), (407-408), (219-220), (408-409), (408-410), (223-237), (185-164), (409-393), (409-394), (237-238), (237-372), (164-165), (165-173), (152-135), (135-145), (135-195), (410-415), (410-411), (411-395), (411-412), (355-354), (354-242), (354-241), (186-166), (412-399), (412-398), (412-413), (166-170), (189-198), (189-193), (170-174), (198-196), (356-357), (174-180), (413-397), (413-396), (357-244), (357-245), (290-291), (289-290), (246-247), (247-248), (266-324), (288-289), (267-266), (300-288), (300-286), (300-301), (300-307), (268-267), (268-327), (268-297), (268-292), (269-268), (359-358), (359-250), (365-269), (365-366), (285-300), (313-317), (366-270), (366-271), (302-303), (360-251), (363-365), (363-381), (253-360), (362-363), (362-272), (254-253), (254-361), (254-255), (308-316), (361-256), (296-260), (296-299), (296-298), (260-287), (364-284), (364-285), (324-326), (324-325), (324-323), (334-335), (283-257), (323-336), (327-328), (338-337), (322-329), (297-296), (339-338), (321-322), (376-233), (299-321), (230-376), (367-340), (340-341), (329-330), (329-333), (371-374), (294-295), (341-342), (341-368), (330-331), (374-227), (374-226), (292-294), (331-332), (226-225), (333-334) |
| **Optimal tree** | (384-68), (68-93), (68-67), (89-88), (67-89), (67-64), (225-224), (224-222), (21-47), (21-50), (72-21), (72-74), (72-76), (3-20), (20-51), (20-44), (222-221), (74-75), (70-59), (382-70), (51-52), (51-54), (221-344), (76-81), (52-56), (220-223), (59-69), (56-27), (1-273), (1-274), (1-123), (1-92), (1-351), (1-350), (1-375), (1-211), (1-215), (1-190), (1-373), (1-362), (1-364), (274-275), (275-276), (73-72), (73-80), (60-58), (40-31), (209-208), (209-210), (80-82), (65-71), (71-63), (47-40), (276-264), (77-73), (69-65), (54-24), (264-263), (264-265), (78-77), (24-29), (263-262), (79-78), (83-79), (63-61), (61-60), (29-34), (262-261), (90-83), (257-259), (257-254), (257-258), (64-62), (44-25), (44-32), (372-239), (239-240), (239-243), (123-120), (25-35), (265-339), (265-367), (120-94), (94-113), (94-111), (94-119), (94-98), (35-38), (369-347), (369-370), (38-41), (370-348), (370-90), (114-115), (115-112), (92-2), (92-87), (41-43), (2-17), (2-14), (2-13), (383-382), (251-383), (251-359), (251-252), (113-114), (17-18), (50-48), (50-46), (208-277), (208-236), (277-278), (277-279), (106-95), (6-7), (33-55), (33-22), (84-33), (84-104), (84-109), (84-110), (84-107), (279-280), (111-106), (18-5), (5-6), (28-36), (36-45), (280-281), (45-49), (119-121), (119-122), (119-200), (235-234), (14-16), (55-23), (23-28), (282-283), (118-84), (8-9), (9-3), (234-282), (122-118), (236-235), (16-19), (19-8), (104-105), (32-37), (232-231), (232-293), (349-232), (37-42), (207-349), (13-4), (380-229), (229-228), (109-108), (4-12), (12-15), (228-371), (231-230), (110-103), (11-86), (30-39), (86-91), (53-26), (22-53), (304-306), (304-378), (97-117), (116-97), (10-11), (26-30), (378-377), (101-100), (96-101), (88-10), (309-314), (309-313), (307-309), (307-312), (100-102), (57-147), (57-149), (57-148), (314-318), (314-310), (317-319), (102-99), (187-167), (167-175), (318-315), (138-194), (117-96), (175-181), (301-305), (301-302), (305-304), (155-138), (151-155), (151-152), (181-178), (310-320), (315-311), (124-126), (124-127), (98-116), (160-129), (180-160), (180-177), (351-379), (129-141), (129-154), (129-151), (350-203), (147-140), (162-169), (203-204), (203-206), (153-132), (141-153), (141-130), (183-162), (379-380), (182-183), (182-186), (252-66), (204-205), (136-142), (243-384), (243-355), (243-356), (243-246), (132-136), (244-85), (206-207), (163-171), (352-414), (352-212), (352-353), (414-385), (414-401), (149-146), (184-163), (184-187), (125-57), (401-386), (401-402), (375-202), (127-128), (128-125), (131-157), (157-133), (402-387), (402-403), (130-131), (211-209), (200-201), (200-158), (403-388), (403-404), (158-124), (404-400), (404-405), (404-407), (368-345), (368-343), (345-346), (345-369), (373-352), (199-192), (199-197), (154-156), (154-134), (197-191), (137-144), (353-213), (353-214), (159-199), (159-161), (159-182), (159-184), (148-150), (156-137), (405-389), (405-406), (190-159), (190-189), (215-216), (161-172), (161-185), (150-143), (406-390), (406-391), (216-217), (216-218), (172-188), (188-168), (134-139), (218-219), (168-176), (407-392), (407-408), (219-220), (176-179), (179-173), (408-409), (408-410), (223-237), (185-164), (409-393), (409-394), (237-238), (237-372), (164-165), (146-145), (152-135), (135-195), (410-415), (410-411), (411-395), (411-412), (355-354), (354-242), (354-241), (186-166), (412-399), (412-398), (412-413), (166-170), (189-198), (189-193), (170-174), (198-196), (356-357), (174-180), (413-397), (413-396), (357-244), (357-245), (290-291), (289-290), (246-247), (247-248), (266-324), (288-289), (267-266), (300-288), (300-286), (300-301), (300-307), (268-267), (268-327), (268-297), (268-292), (358-249), (269-268), (359-358), (359-250), (365-269), (365-366), (285-300), (313-317), (366-270), (366-271), (302-303), (360-251), (363-365), (363-381), (253-360), (362-363), (362-272), (254-253), (254-361), (254-255), (361-256), (316-308), (296-260), (296-299), (296-298), (260-287), (312-316), (364-284), (364-285), (324-326), (324-325), (324-323), (334-335), (283-257), (323-336), (327-328), (338-337), (322-329), (297-296), (339-338), (321-322), (376-233), (299-321), (230-376), (367-340), (340-341), (329-330), (329-333), (371-374), (294-295), (341-342), (341-368), (330-331), (374-227), (374-226), (292-294), (331-332), (226-225), (333-334) |
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